TIMOTHY
CHRISTIAN SCHOOL

Entering AP Calculus AB/BC Summer Packet

- *All students approved for AP Calculus AB or BC are to complete this packet. BC material is labeled as such and does not need to be completed by AB students.*

- *The packet includes links to Office 365 FORMS quizzes and IXL skills. Therefore, students will need to access their **Timothy Christian School** Office 365 and IXL accounts.*

- *Unless told otherwise, this summer assignment will be due on the first day of class and will be graded on neatness, completeness and accuracy.*

- *Students may use external resources to assist them with these problems. However, all work submitted must be done by the student. Show all your logical steps in arriving at the solution.* ***Listing only answers will not earn a student any credit.*** *For the most advanced topics (Introductory Calculus topics) solutions are provided.*

- ***Students are to be prepared for a quiz on this content the first full week of school.***

Pre-requisite skills needed for AP Calculus

I. Algebra

A. Exponents (operations with integer, fractional, and negative exponents)

B. Factoring (GCF, trinomials, difference of squares and cubes, sum of cubes, grouping)

C. Rationalizing (denominator)

D. Simplifying rational expressions

E. Solving algebraic equations and inequalities

F. Simultaneous equations

II. Graphing Functions

A. Lines (intercepts, slopes, write equations using point slope and slope intercept form, parallel, perpendicular, distance and midpoint formulas)

B. Conic Sections (circle, parabola, ellipse, hyperbola)

C. Functions (definition, notation, domain, range, inverse, composition)

D. Basic shapes and transformations of the following functions (absolute value, rational, root, higher order curves, log, ln, exponential, trigonometric, piece wise, inverse functions)

E. Tests for symmetry: odd, even

III. Logarithmic and Exponential Functions

A. Simplify expressions

B. Solve exponential and logarithmic equations

C. Inverses

IV. Geometry and Trigonometry

A. Area and Perimeter

B. Unit Circle (definition of functions, angles in radians and degrees)Unit Circle

C. Trig Functions

D. Identities

E. Solving Trig Equations

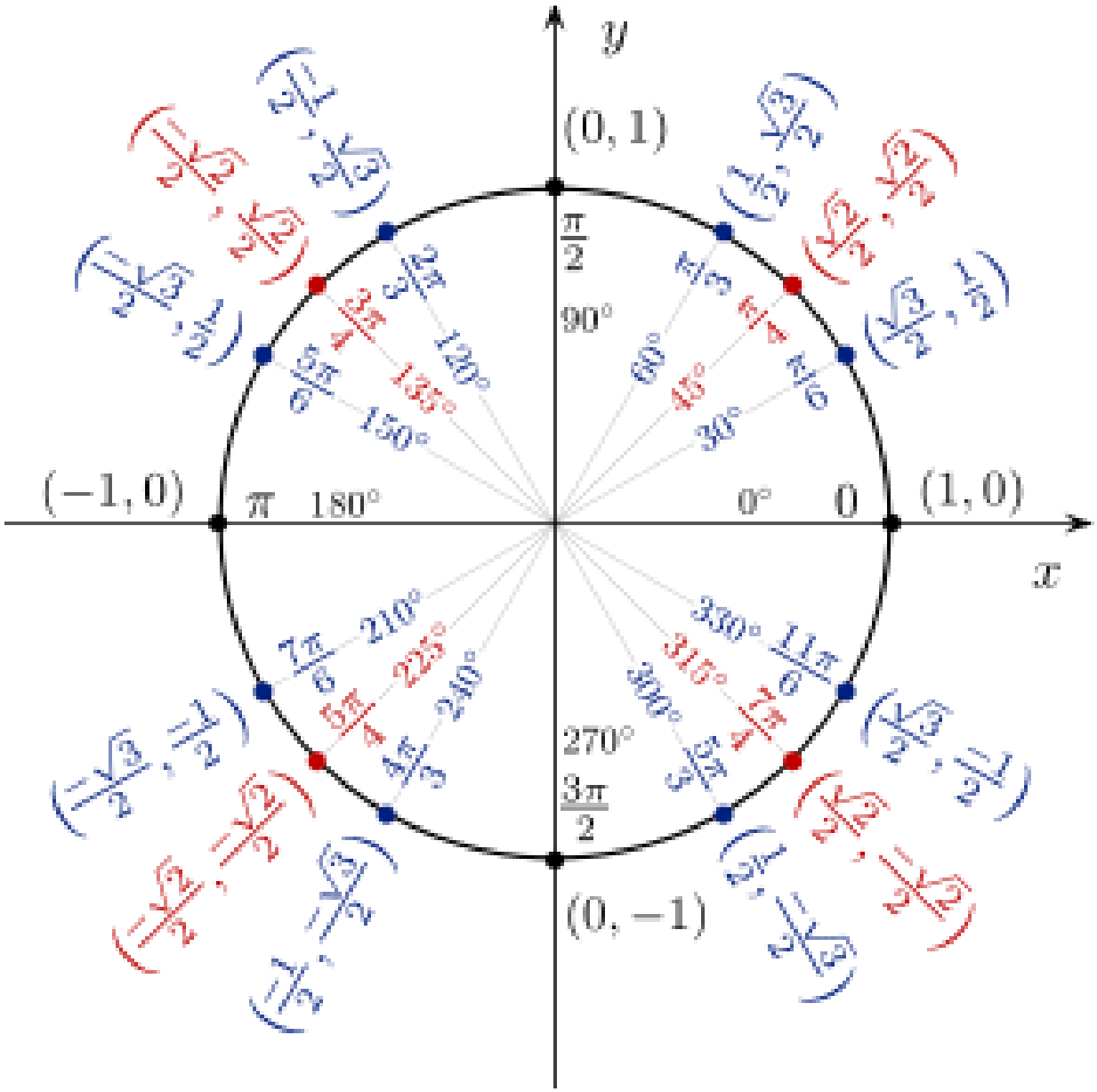
F. Inverse Trigonometric Functions

G. Right Triangle Trigonometry

V. Graphing Calculator

UNIT CIRCLE

RADIUS = 1 UNIT



Calculus Prerequisite Problems

I. Algebra

A. Simplify $\dfrac{(8x^3yz)^{\frac{1}{3}}(2x)^3}{4x^{\frac{1}{3}}(yz^{\frac{2}{3}})^{-1}}$

B. Factor Completely (Grouping, GCF, difference of squares or cubes)

1. $9x^2 + 3x - 3xy - y$

2. $64x^6 - 1$

3. $42x^4 + 35x^2 - 28$

4. $15x^{\frac{5}{2}} - 2x^{\frac{3}{2}} - 24x^{\frac{1}{2}}$ $(factor\ out\ x^{\frac{1}{2}}\ first)$

5. Factor out 4 from this expression: $16x^6 - 4$

6. Factor out 4 from this expression: $x^6 - 4$

C. Rationalize and simplify where possible.

1. $\frac{3-x}{1-\sqrt{x-2}}$

2. $\frac{x^2-3}{x-\sqrt{3}}$

3. *Explain the error:* $\sqrt{x^2+4} = x + 2$

D. Simplfy the rational expression

1. $\frac{(x+1)^3(x-2)+3(x+1)^2}{(x+1)^4}$

2. *Explain the error:* $\frac{x^2-1}{x-1} = x$

E. Solve algebraic equations and inequalities

1 − 2 *Use synthetic division to help factor the following, state all factors and roots.*

1. $p(x) = x^3 + 4x^2 + x - 6$

2. $p(x) = 6x^3 - 17x^2 - 16x + 7$

Rewrite the function F below in terms of the given variable.

3. x + y = 10; F= xy F(x) =?

4 $x^2 + y^2 = 25$ $F = xy$ Find F(x)

Solve: Use a graphing calculator to check solutions.

5. $(x+3)^2 > 4$

6. $\dfrac{x+5}{x-3} \leq 0$

7. $3x^3 - 14x^2 - 5x \leq 0$ $(factor)$

8. $x < \dfrac{1}{x}$

9. $\dfrac{x^2-9}{x+1} \geq 0$

10. $\dfrac{1}{x-1} + \dfrac{4}{x-6} > 0$

11. $x^2 < 4$

12. $|2x+1| < \dfrac{1}{4}$

F. Solve the system. Solve the system algebraically and then check the solution by graphing each function and using your calculator to find the points of intersection.

1. $\begin{matrix} x - y + 1 = 0 \\ y - x^2 = -5 \end{matrix}$

2. $\begin{matrix} x^2 - 4x + 3 = y \\ -x^2 + 6x - 9 = y \end{matrix}$

II. Graphing and Functions:

A. Linear graphs: Write the equation of the line described below.

1. *Passes through the point* $(2, -1)$ *and has a slope of* $-\frac{1}{3}$

2. *Passes through the point* $(4, -3)$ *and is perpendicular to* $3x + 2y = 4$.

3. *Passes through* $(-1, -2)$ *and is parallel to* $y = \frac{3}{5}x - 1$

B. Conic Sections: Write the equation in standard form and identify the conic.

$4x^2 - 16x + 3y^2 + 24y + 52 = 0$

C. Functions: Find the domain and range of the following.

Note: domain restrictions – denominator $\neq 0$, *argument of a log or ln* > 0,
radicand of even index must be ≥ 0

range restrictions – reasoning, if all else fails, use graphing calculator

1. $y = \frac{3}{x-2}$

2. $\log(x - 3)$

3. $y = x^4 + x^2 + 2$

4. $y = \sqrt{2x - 3}$

5. $y = |x - 5|$

6. $y = \frac{\sqrt{x+1}}{x^2-1}$ *domain only*

7. *Given* $f(x)$ *below, graph over the domain* $[-3, 3]$, *what is the range?*

$$f(x) = \begin{cases} x & if\ x \geq 0 \\ 1 & if\ -1 \leq x < 0 \\ x - 2 & x < -1 \end{cases}$$

Find the composition /inverse as indicated below.

Let $f(x) = x^2 + 3x - 2$ $\quad g(x) = 4x - 3$ $\quad h(x) = lnx$ $\quad w(x) = \sqrt{x - 4}$

8. $g^{-1}(x)$ 9. $h^{-1}(x)$ 10. $w^{-1}(x), for\ x \geq 4$ 11. $f\big(g(x)\big)$ 12. $h(g(f(1)))$

13. *Does* $y = 3x^2 - 9$ *have an inverse function? Explain your answer.*

Let $f(x) = 2x, \quad g(x) = -x, \quad and\ h(x) = 4, find$

14. $(f\ o\ g)(x)$ 15. $h(g(x))$ 16. $g(h(x))$ 17. *Slope of* $h(x)$.

18. f(3) 19. g(3) 20. $(f\ o\ g)(3)$ 21. Slope of $g(h(x))$.

D. Basic Shapes of Curves:

Sketch the graphs. You may use your graphing calculator to verify your graph, but you should be able to graph the following by knowledge of the shape of the curve, by plotting a few points, and by your knowledge of transformations.

1. $y = \sqrt{x}$ 2. $y = lnx$ 3. $y = \frac{1}{x}$ 4. $y = |x - 2|$ 5. $y = \frac{1}{x-2}$

6. $y = \frac{x}{x^2-4}$ 7. $y = 2^{-x}$ 8. $y = 3sin2(x - \frac{\pi}{6})$

9. $f(x) = \begin{cases} \sqrt{25 - x^2} & if\ x < 0 \\ \frac{x^2 - 25}{x - 5} & if\ x \geq 0,\ x \neq 5 \\ 0 & if\ x = 5 \end{cases}$

E. Even, Odd, Tests for Symmetry:

Identify as odd, even , or neither and justify you're answer. To justify your answer you must show substitution using $-x$ *! It is not enough to simply check a number.*

Even: *if* $f(x) = f(-x)$ *Odd*: *if* $f(-x) = -f(x)$

1. $f(x) = x^3 + 3x$ 2. $f(x) = x^4 - 6x^2 + 3$ 3. $f(x) = \frac{x^3 - x}{x^2}$

4. $f(x) = sin\ 2x$ 5. $f(x) = x^2 + x$ 6. $f(x) = x(x^2 - 1)$

7. $f(x) = \frac{1 + |x|}{x^2}$

8. *What type of function (even or odd) results from the product of two even functions?*

Two odd functions?

Test for symmetry. Show substitution with variables to justify you're answer.

→ *Symmetric to y axis: replace x with* $-$ *x and relation remains the same.*

→ *Symmetric to x axis: replace y with* $-$ *y and relation remains the same.*

→ *Origin symmetry: replace x with* $-$ x, y *with* $-$ *y and the relation is equivalent.*

1. $y = x^4 + x^2$ 2. $y = sin(x)$ 3. $y = cos(x)$

4. $x = y^2 + 1$ 5. $y = \dfrac{|x|}{x^2 + 1}$

III. Logarithmic and Exponential Functions

A. Simplify expressions:

1. $log_4\left(\frac{1}{16}\right)$ 2. $3log_3 3 - \frac{3}{4}log_3 81 + \frac{1}{3}log_3\left(\frac{1}{27}\right)$ 3. $log_9 27$

4. $log_{125}\left(\frac{1}{5}\right)$ 5. $log_w w^{45}$ 6. $ln\, e$ 7. $\ln 1$ 8. lne^2

B. Solve equations:

1. $log_6(x + 3) + log_6(x + 4) = 1$ 2. $log\, x^2 - log\, 100 = log\, 1$ 3. $3^{x+1} = 15$

C. *Find the inverse for each function.*

a. $f(x) = \sqrt{10 - 3x}$

b. $f(x) = e^{x^3}$

c. $y = \ln(x + 3)$

IV. Geometry and Trigonometry

A. Area and Perimeter

1. *A rectangle has perimeter* 20 *m. Express the area of the rectangle as a function of the length of one of its sides.*

2. Find the area of a 30 degree sector in a circle with radius equal to 5 inches.

B. Unit Circle: Know the unit circle $-$ *radian and degree measure. Be prepared for a quiz.*

C. Trigonometric Functions

1. *State the domain, range and period for each function*?

a) $y = sin\, x$ *b)* $y = cos\, x$ *c)* $y = tan\, x$

D. Identities:

Simplify: 1. $\dfrac{(tan^2x)(csc^2x) - 1}{(cscx)(tan^2x)(sinx)}$ 2. $1 - cos^2x$ 3. $sec^2x - tan^2x$

Verify: 4. $(1 - sin^2x)(1 + tan^2x) = 1$

E. Solve the Equations

1. $cos^2x = cos\,x + 2, \quad 0 \le x \le 2\pi$ 2. $2\,sin(2x) = \sqrt{3}, \quad 0 \le x \le 2\pi$

3. $cos^2x + sinx + 1 = 0, \quad 0 \le x \le 2\pi$

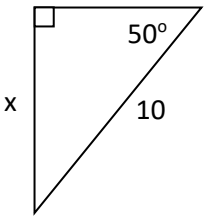
F. Inverse Trig Functions: Note: $Sin^{-1}x = Arcsin\,x$

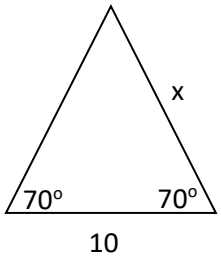
1. $Arcsin1$ 2. $Arcsin\left(-\dfrac{\sqrt{2}}{2}\right)$ 3. $Arccos\left(\dfrac{\sqrt{3}}{2}\right)$ 4. $sin\left(Arccos\left(\dfrac{\sqrt{3}}{2}\right)\right)$

5. *State domain and range for*: $Arcsin(x)$, $Arccos\,(x)$, $Arctan\,(x)$

G. Right Triangle Trig: Find the value of x. (Note: Degree measure!)

1. 2.





***V*. *Graphing Calculator*:**

Be familiar with the $\boxed{CALC}$ *commands; value, root, minimum, maximum, intersect. You may need to zoom in on areas of your graph to find the information.*

Answers should be accurate to 3 *decimal places. Sketch graph.*

1 − 4 *Given the following function* $f(x) = 2x^4 - 11x^3 - x^2 + 30x$.

1. *Find all roots.* *Note: Window x min:* -10 *x max:* 10 *scale* 1 *y min:* -100 *y max:* 60 *scale* 10
2. *Find all local maxima.*
3. *Find all local minima.*
4. *Find the following values:* $f(-1), f(2), f(0), f(.125)$

A local maximum or local minimum is a point on the graph where there is a highest or lowest poin within an interval such as the vertex of a parabola.

5. *Graph the following two functions and find their points of intersection using the intersect command on your calculator.*

$y = x^3 + 5x^2 - 7x + 2$ *and* $y = 0.2x^2 + 10$ *Window: x min :* -10 *x max:* 10 *scale* 1 *y min:* -10 *y max:* 50 *scale* 0

6. *Use a graphing calculator to determine which of the given viewing rectangles produces the most appropriate graph of the function* $3\,f(x) = 10 + 25x - x^3$.

a. $[-4,4]$ *by* $[-4,4]$

b. $[-10,10]$ *by* $[-10,10]$

c. $[-20,20]$ *by* $[-100,100]$

d. $[-100,100]$ *by* $[-200,200]$

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Introductory Calculus

A)Limits

1. Complete this form: Limits and Continuity Practice Test – Fill out form

2. Evaluate using numerical methods. The angle x must be in radians.

$$\lim_{x \to 0} \frac{\sin(x)}{x}$$

3. Evaluate using numerical methods. The angle x must be in radians.

$$\lim_{x \to 0} \frac{\cos x - 1}{x}$$

3. Using the limits in problems 2 and 3 and the sum and difference identities, show that the **derivative of sin (x)** = cos (x). FILL IN THE FORM. Proof of sin (x) derivative – Fill out form

Then practice writing the proof by hand. You will be quizzed on this proof the first week of class.

STEP1. Write the limit of the difference quotient.

STEP2. Rewrite sin(x+h) using sum identity.

STEP3. Rearrange terms in the numerator and split the fraction to see the special limits.

STEP4. Evaluate by distributing the limit into each factor of each term.

4. The derivative of cos(x) = -sin(x). You will need this to complete problems below. Practice this proof as well. You are expected to Google it or use AI. Verify all guidance provided by AI with multiple sources.

B) Limit of the Difference Quotient: Derivative

1) Simplify using the fact that $e^c \approx 1 + c \ \ for\ c \ \approx 0$ And use logarithm properties also in part b.

a) $\lim_{h\to 0} \frac{e^{x+h} - e^x}{h}$

b) $\lim_{h\to 0} \frac{\ln(x+h) - \ln(x)}{h}$ Use logarithm properties also.

2) $\lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$ for

a) $f(x) = \sqrt{x}$; Hint: Multiply numerator and denominator by the conjugate.

b) $f(x) = x^4$

c) $f(x) = x^4 + 2$

d) $f(x) = x^4 + 2x$

3) Use the limit of the difference quotient to find the derivative function f'(x) for the following functions. Write your answers in terms of u'(x), v'(x), and w'(x).

a) $f(x) = u(x) + v(x)$

b) $f(x) = 2u(x)$

c) $f(x) = u(x) + v(x) + w(x)$

d) $f(x) = u(x) - v(x)$

e) $f(x) = 2u(x) + 3v(x)$

f) $f(x) = \frac{1}{3}u(x) - 5v(x)$

g) $f(x) = \frac{1}{3}x + 2x^2 - 3x^3$

What rule can you make about finding the derivative of a sum?'

C) Tangent Line Equations Worksheet (followed by answer keys)

Part 1: Building Tangent Line Equations (No Calculating)

Directions: Write the equation of the tangent line using the information given. Do not calculate derivatives or slopes.

1. The tangent line touches the curve at x = 2.

   f(2) = 5, f'(2) = -3

2. A function g(x) has a tangent line at x = -1.

   g(-1) = 4, g'(-1) = 6

3. At x = 0, a function satisfies:

   f(0) = -2, f'(0) = 1

4. A curve has a tangent line at x = 3 with:

   Point on the curve: (3, 7)

   Slope of tangent line: -1/2

5. The tangent line to a function at x = a has:

   f(a) = b, f'(a) = m

Part 2: Simple Derivatives and Tangent Lines

Directions:

- **Find the derivative.**
- **Evaluate the derivative at the given x-value.**
- **Write the equation of the tangent line.**

1. $f(x) = x^2$ at x = 1

2. $g(x) = 3x - 5$ at x = 2

3. $h(x) = x^3$ at x = -1

4. $p(x) = 4x^2 - x$ at x = 0

5. $q(x) = \frac{1}{2}x^2$ at x = 4

Part 3: Transcendental Functions

Directions: Find the equation of the tangent line at the given value of x.

1. $f(x) = e^x$ at x = 0

2. $g(x) = lnx$ at x = 1

3. $h(x) = sinx$ at x = 0

4. $p(x) = cosx$ at x = π/2

5. $q(x) = e^{2x}$ at x = 0 You may google the derivative for e^{2x}.

D) Linearization & Tangent Line Approximation Practice

LEVEL 1: Approximate Using the Linearization

Goal: Use $L(x)$to approximate nearby values.

1. Let $f(x) = x^2$. Use the tangent line at $x = 3$ to approximate:

$$(3.1)^2$$

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2. Let $g(x) = \sqrt{x}$. Use the tangent line at $x = 4$ to approximate:

$$\sqrt{3.9}$$

---

3. Let $h(x) = e^x$. Use the tangent line at $x = 0$to approximate:

$$e^{0.05}$$

LEVEL 2: Trig & Log Linearization (AP-Style)

Goal: Apply linearization to non-polynomial functions.

4.Use a linearization to approximate: $\sin(0.1)$

by linearizing $f(x) = \sin x$at $x = 0$.

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5. Use a linearization to approximate: $\ln(1.02)$

by linearizing $f(x) = \ln x$at $x = 1$.

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6. Use a linearization to approximate: $\cos(0.05)$

by linearizing $f(x) = \cos x$at $x = 0$.

E) Complete the IXL skills and FORMS quizzes.

Using your TCS IXL account, complete assigned (starred) skills. Achieve a Smartscore of at least 80. YOU MUST SIGN IN USING YOUR TCS ACCOUNT.

JJ. Introduction to derivatives

- 1 Average rate of change I
- 2 Average rate of change II
- 3 Find instantaneous rates of change
- 4 Velocity as a rate of change
- 5 Find values of derivatives using limits
- 6 Find the slope of a tangent line using limits
- 7 Find equations of tangent lines using limits

Using your Microsoft 365 TCS Account, complete the FORMs quizzes.

Linearization- NO CALCULATOR ALLOWED – Fill out form

Notes:

Trig Drill AP CALC – Fill out form

Notes:

F) (BC Only) Sequences and Series:

Using your TCS IXL account, complete assigned (starred) skills. Achieve a Smartscore of at least 80. YOU MUST SIGN IN USING YOUR TCS ACCOUNT.

AA. Sequences

- ★ 1 Find terms of a sequence
- ★ 2 Find terms of a recursive sequence
- ★ 3 Identify a sequence as explicit or recursive
- ★ 4 Find a recursive formula
- ★ 5 Find recursive and explicit formulas
- ★ 6 Convert a recursive formula to an explicit formula
- ★ 7 Convert an explicit formula to a recursive formula
- ★ 8 Convert between explicit and recursive formulas

BB. Series

- ★ 1 Identify arithmetic and geometric series
- ★ 2 Introduction to sigma notation
- ☆ 3 Find the sum of an arithmetic series
- ★ 4 Find the sum of a finite geometric series
- ★ 5 Introduction to partial sums
- ☆ 6 Partial sums of arithmetic series
- ★ 7 Partial sums of geometric series
- ★ 8 Partial sums: mixed review
- ★ 9 Convergent and divergent geometric series
- ★ 10 Find the value of an infinite geometric series
- ★ 11 Write a repeating decimal as a fraction

Tangent Line Equations – Answer Key

Part 1: Building Tangent Line Equations

1. $y - 5 = -3(x - 2)$

2. $y - 4 = 6(x + 1)$

3. $y + 2 = 1(x - 0)$

4. $y - 7 = -1/2(x - 3)$

5. $y - b = m(x - a)$

Part 2: Simple Derivatives and Tangent Lines

1. $f(x) = x^2$; $f'(x) = 2x$; $f'(1) = 2, f(1) = 1$; $y - 1 = 2(x - 1)$

2. $g(x) = 3x – 5$; $g'(x) = 3$; $g'(2) = 3, g(2) = 1$; $y - 1 = 3(x - 2)$

3. $h(x) = x^3$; $h'(x) = 3x^2$; $h'(-1) = 3, h(-1) = -1$; $y + 1 = 3(x + 1)$

4. $p(x) = 4x^2 – x$; $p'(x) = 8x – 1$; $p'(0) = -1, p(0) = 0$; $y = -x$

5. $q(x) = (1/2)x^2$; $q'(x) = x$; $q'(4) = 4, q(4) = 8$; $y - 8 = 4(x - 4)$

Part 3: Transcendental Functions

1. $f(x) = e^x$; $f'(x) = e^x$; $f'(0) = 1, f(0) = 1$; $y - 1 = x$

2. $g(x) = \ln x$; $g'(x) = 1/x$; $g'(1) = 1, g(1) = 0$; $y = x - 1$

3. $h(x) = \sin x$; $h'(x) = \cos x$; $h'(0) = 1, h(0) = 0$; $y = x$

4. $p(x) = \cos x$; $p'(x) = -\sin x$; $p'(\pi/2) = -1, p(\pi/2) = 0$; $y = -(x - \pi/2)$

5. $q(x) = e^{(2x)}$; $q'(x) = 2e^{(2x)}$; $q'(0) = 2, q(0) = 1$; $y - 1 = 2x$

Linearization & Tangent Line Approximation Practice – Answer Key

Level 1 Answers

1. $L(x) = 9 + 6(x - 3)$

Approximate: $L(3.1) = 9 + 6(0.1) = 9.6$

$(3.1)^2 \approx 9.6$

2. $L(x) = 2 + \frac{1}{4}(x - 4)$

Approximate: $L(3.9) = 2 + \frac{1}{4}(-0.1) = 1.975$

$\sqrt{3.9} \approx 1.975$

3. $L(x) = 1 + x$

Approximate: $L(0.05) = 1.05$

$e^{0.05} \approx 1.05$

Level 3 Answers

4. $f(x) = \sin x, f'(x) = \cos x$

$f(0) = 0, f'(0) = 1$

$L(x) = x$

$\sin(0.1) \approx 0.1$

5. $f(x) = \ln x, f'(x) = \frac{1}{x}$

$f(1) = 0, f'(1) = 1$

$L(x) = x - 1$

$\ln(1.02) \approx 0.02$

6. $f(x) = \cos x, f'(x) = -\sin x$

$f(0) = 1, f'(0) = 0$

$L(x) = 1$

$\cos(0.05) \approx 1$