



TIMOTHY
CHRISTIAN SCHOOL

Entering AP Calculus Summer Packet

All students approved for AP Calculus are to complete this packet. Unless told otherwise, this summer assignment will be due on the first day of class and will be graded on neatness, completeness and accuracy. Show all your logical steps in arriving at the solution. Do not list only answers.

Skills needed for AP Calculus

I. Algebra

- A. Exponents (operations with integer, fractional, and negative exponents)
- B. Factoring (GCF, trinomials, difference of squares and cubes, sum of cubes, grouping)
- C. Rationalizing (denominator)
- D. Simplifying rational expressions
- E. Solving algebraic equations and inequalities
- F. Simultaneous equations

II. Graphing Functions

- A. Lines (intercepts, slopes, write equations using point slope and slope intercept form, parallel, perpendicular, distance and midpoint formulas)
- B. Conic Sections (circle, parabola, ellipse, hyperbola)
- C. Functions (definition, notation, domain, range, inverse, composition)
- D. Basic shapes and transformations of the following functions (absolute value, rational, root, higher order curves, log, ln, exponential, trigonometric, piece wise, inverse functions)
- E. Tests for symmetry: odd, even

III. Logarithmic and Exponential Functions

- A. Simplify expressions
- B. Solve exponential and logarithmic equations
- C. Inverses

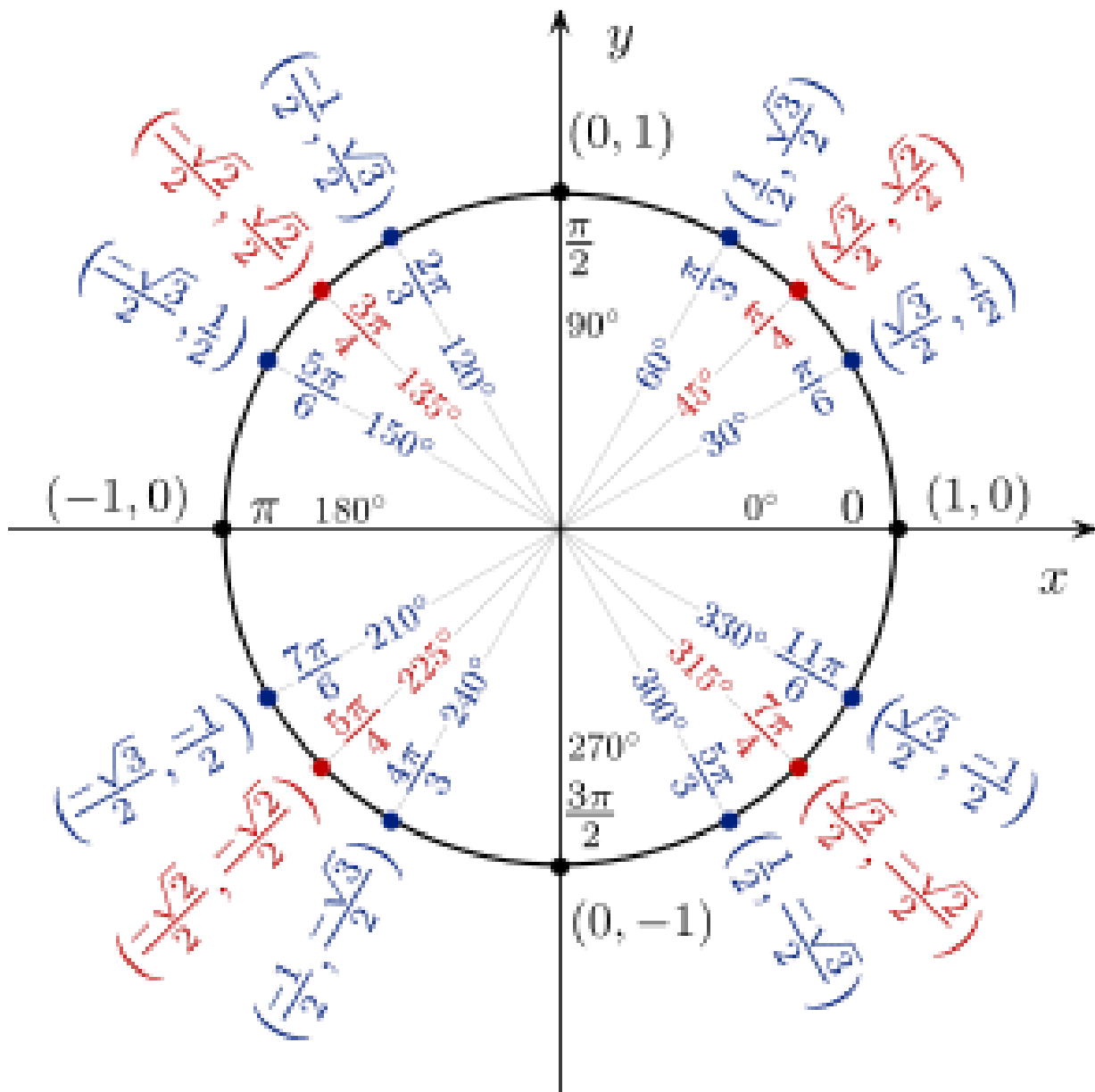
IV. Geometry and Trigonometry

- A. Area and Perimeter
- B. Unit Circle (definition of functions, angles in radians and degrees)Unit Circle
- C. Trig Functions
- D. Identities
- E. Solving Trig Equations
- F. Inverse Trigonometric Functions
- G. Right Triangle Trigonometry

V. Graphing Calculator

UNIT CIRCLE

RADIUS = 1 UNIT



Calculus Prerequisite Problems

I. Algebra

A. Exponent Rules: Simplify the expressions.

1. $\frac{(8x^3yz)^{\frac{1}{3}}(2x)^3}{4x^{\frac{1}{3}}(yz^{\frac{2}{3}})^{-1}}$ 2. $1/2^{-1}$

B. Factoring

Factor Completely (Grouping, GCF, difference of squares or cubes)

1. $9x^2 + 3x - 3xy - y$ 2. $64x^6 - 1$
3. $42x^4 + 35x^2 - 28$ 4. $15x^{\frac{5}{2}} - 2x^{\frac{3}{2}} - 24x^{\frac{1}{2}}$ (factor out $x^{\frac{1}{2}}$ first)
5. Factor out 3 from both terms and rewrite as a product: $16x^6 - 3$
6. Write $x^6 - 4$ as a product of 4 and a binomial: $4(a - b)$

C. Rationalize and simplify where possible.

1. $\frac{3-x}{1-\sqrt{x-2}}$ 2. $\frac{x^2-3}{x-\sqrt{3}}$ 3. Explain the error: $\sqrt{x^2+4} = x+2$

D. Rational Expressions

1. Select the expressions that are equal to $\left(-\frac{2b}{3}\right)$
- a. $(2b)^{-\frac{1}{3}}$ b. $(-2b)^{\frac{1}{3}}$ c. $(b)\left(-\frac{2}{3}\right)$ d. $(2)\left(-\frac{b}{3}\right)$ e. $(b)\left(-\frac{1/3}{1/2}\right)$ f. All are equivalent
2. $\frac{(x+1)^3(x-2)+3(x+1)^2}{(x+1)^4}$ 3. Explain the error: $\frac{x^2-1}{x-1} = x$

E. Solve algebraic equations and inequalities

State all factors and roots. Use synthetic division to help factor.

1. $p(x) = x^3 + 4x^2 + x - 6$ 2. $p(x) = 6x^3 - 17x^2 - 16x + 7$

Rewrite the second function below in terms of the given variable.

3. $x + y = 10$; $F = xy$ Find $F(x)$. 4. $x^2 + y^2 = 25$ $F = xy$ Find $F(x)$.

Solve: Use a graphing calculator to check solutions.

5. $(x + 3)^2 > 4$
6. $\frac{x + 5}{x - 3} \leq 0$
7. $3x^3 - 14x^2 - 5x \leq 0$ (factor)
8. $x < \frac{1}{x}$
9. $\frac{1}{x - 1} + \frac{4}{x - 6} > 0$
10. $|2x + 1| < \frac{1}{4}$

F. Solve the system. Solve the system algebraically and then check the solution by graphing each function and using your calculator to find the points of intersection.

1. $\begin{cases} x - y + 1 = 0 \\ y - x^2 = -5 \end{cases}$
2. $\begin{cases} x^2 - 4x + 3 = y \\ -x^2 + 6x - 9 = y \end{cases}$

II. Graphing and Functions:

A. Linear graphs: Write the equation of the line described below.

1. Passes through the point $(2, -1)$ and has a slope of $-\frac{1}{3}$
2. Passes through the point $(4, -3)$ and is perpendicular to $3x + 2y = 4$.
3. Passes through $(-1, -2)$ and is parallel to $y = \frac{3}{5}x - 1$

B. Conic Sections: Write the equation in standard form and identify the conic.

$$4x^2 - 16x + 3y^2 + 24y + 52 = 0$$

C. Functions: Find the domain and range of the following.

Note: domain restrictions – denominator $\neq 0$, argument of a log or $\ln > 0$,

radicand of even index must be ≥ 0

range restrictions – reasoning, if all else fails, use graphing calculator

1. $y = \frac{3}{x - 2}$
2. $\log(x - 3)$
3. $y = x^4 + x^2 + 2$
4. $y = \sqrt{2x - 3}$
5. $y = |x - 5|$
6. $y = \frac{\sqrt{x+1}}{x^2-1}$ domain only

7. Given $f(x)$ below, graph over the domain $[-3, 3]$, what is the range?

$$f(x) = \begin{cases} x & \text{if } x \geq 0 \\ 1 & \text{if } -1 \leq x < 0 \\ x - 2 & \text{if } x < -1 \end{cases}$$

Find the composition /inverse as indicated below.

$$\text{Let } f(x) = x^2 + 3x - 2 \quad g(x) = 4x - 3 \quad h(x) = \ln x \quad w(x) = \sqrt{x - 4}$$

8. $g^{-1}(x)$ 9. $h^{-1}(x)$ 10. $w^{-1}(x)$, for $x \geq 4$ 11. $f(g(x))$ 12. $h(g(f(1)))$
 13. Does $y = 3x^2 - 9$ have an inverse function? Explain your answer.

Let $f(x) = 2x$, $g(x) = -x$, and $h(x) = 4$, find

14. $(f \circ g)(x)$ 15. $h(g(x))$ 16. $g(h(x))$ 17. Slope of $h(x)$.
 18. $f(3)$ 19. $g(3)$ 20. $(f \circ g)(3)$ 21. Slope of $g(h(x))$.

D. Basic Shapes of Curves:

Sketch the graphs. You may use your graphing calculator to verify your graph, but you should be able to graph the following by knowledge of the shape of the curve, by plotting a few points, and by your knowledge of transformations.

1. $y = \sqrt{x}$ 2. $y = \ln x$ 3. $y = \frac{1}{x}$ 4. $y = |x - 2|$ 5. $y = \frac{1}{x - 2}$
 6. $y = \frac{x}{x^2 - 4}$ 7. $y = 2^{-x}$ 8. $y = 3\sin\left(2\left(x - \frac{\pi}{6}\right)\right)$

$$9. f(x) = \begin{cases} \sqrt{25 - x^2} & \text{if } x < 0 \\ \frac{x^2 - 25}{x - 5} & \text{if } x \geq 0, x \neq 5 \\ 0 & \text{if } x = 5 \end{cases}$$

E. Even, Odd, Tests for Symmetry:

Identify as odd, even, or neither and justify your answer. To justify your answer you must show substitution using $-x$! It is not enough to simply check a number.

Even: if $f(x) = f(-x)$ Odd: if $f(-x) = -f(x)$

1. $f(x) = x^3 + 3x$ 2. $f(x) = x^4 - 6x^2 + 3$ 3. $f(x) = \frac{x^3 - x}{x^2}$
 4. $f(x) = \sin 2x$ 5. $f(x) = x^2 + x$ 6. $f(x) = x(x^2 - 1)$
 7. $f(x) = \frac{1 + |x|}{x^2}$

8. What type of function (even or odd) results from the product of two even functions?

Two odd functions?

Test for symmetry. Show substitution with variables to justify your answer.

→ **Symmetric to y axis: replace x with $-x$ and relation remains the same.**

→ **Symmetric to x axis: replace y with $-y$ and relation remains the same.**

→ **Origin symmetry: replace x with $-x$, y with $-y$ and the relation is equivalent.**

1. $y = x^4 + x^2$

2. $y = \sin(x)$

3. $y = \cos(x)$

4. $x = y^2 + 1$

5. $y = \frac{|x|}{x^2 + 1}$

III. Logarithmic and Exponential Functions

A. Simplify expressions:

1. $\log_4\left(\frac{1}{16}\right)$

2. $3\log_3 3 - \frac{3}{4}\log_3 81 + \frac{1}{3}\log_3\left(\frac{1}{27}\right)$

3. $\log_9 27$

4. $\log_{125}\left(\frac{1}{5}\right)$

5. $\log_w w^{45}$

6. $\ln e$

7. $\ln 1$

8. $\ln e^2$

B. Solve for x:

1. $\log_6(x + 3) + \log_6(x + 4) = 1$

2. $\log x^2 - \log 100 = \log 1$

3. $3^{x+1} = 15$

C. Find the inverse for each function.

a. $f(x) = \sqrt{10 - 3x}$

b. $f(x) = e^{x^3}$

c. $y = \ln(x + 3)$

IV. Geometry and Trigonometry

A. Area and Perimeter

1. A rectangle has perimeter 20 m. Express the area of the rectangle as a function of the length of one of its sides.

2. Find the area of a 30 degree sector in a circle with radius equal to 5 inches.

B. Unit Circle: Know the unit circle – radian and degree measure. Create flashcards or quizlet!

Know the sine and cosine values of 0 , $\frac{\pi}{2}$, π , $\frac{3\pi}{2}$ and 2π . Know the tangent of 0 , $\frac{\pi}{4}$, and $\frac{\pi}{2}$.

C. Trigonometric Functions

1. State the domain, range and period for each function?

a) $y = \sin x$

b) $y = \cos x$

c) $y = \tan x$

D. Identities:

Simplify: 1. $\frac{(\tan^2 x)(\csc^2 x) - 1}{(\csc x)(\tan^2 x)(\sin x)}$ 2. $1 - \cos^2 x$ 3. $\sec^2 x - \tan^2 x$

Verify: 4. $(1 - \sin^2 x)(1 + \tan^2 x) = 1$

E. Solve the Equations

1. $\cos^2 x = \cos x + 2$, $0 \leq x \leq 2\pi$ 2. $2 \sin(2x) = \sqrt{3}$, $0 \leq x \leq 2\pi$

3. $\cos^2 x + \sin x + 1 = 0$, $0 \leq x \leq 2\pi$

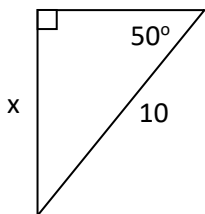
F. Inverse Trig Functions: Note: $\sin^{-1} x = \arcsin x$

1. $\arcsin 1$ 2. $\arcsin\left(-\frac{\sqrt{2}}{2}\right)$ 3. $\arccos\left(\frac{\sqrt{3}}{2}\right)$ 4. $\sin\left(\arccos\left(\frac{\sqrt{3}}{2}\right)\right)$

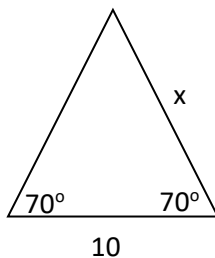
5. State domain and range for: $\arcsin(x)$, $\arccos(x)$, $\arctan(x)$

G. Right Triangle Trig: Find the value of x . (Note: Degree measure!)

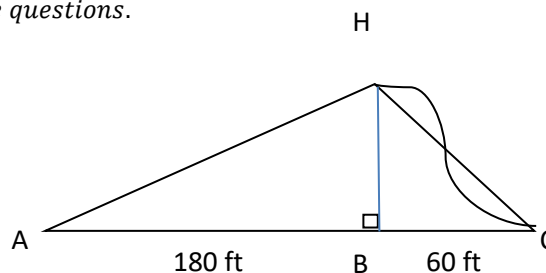
1.



2.



3. A roller coaster car travels as shown in the diagram below. The car covers horizontal distances of 180 feet on the incline and 60 feet on the drop. Answer the questions.



- How high is the roller coaster above point B?
- Find the distances AH and HC (straight line segments).
- The car travels from A to H in 2.8 secs. How fast (in ft/sec) does the car go up the incline?
- The car travels from H to C in 1.2 secs.
Find its average speed assuming it traveled along the straight segment HC?
- If the car actually traveled along the curved track between HC in 1.2 sec, is the actual speed faster or slower?

V. Graphing Calculator:

Be familiar with the **CALC** commands; value, root, minimum, maximum, intersect. You may need to zoom in on areas of your graph to find the information.

Answers should be accurate to 3 decimal places. Sketch graph.

1 - 4 Given the following function $f(x) = 2x^4 - 11x^3 - x^2 + 30x$.

- Find all roots. Note: Window x min: -10 x max: 10 scale 1 y min: -100 y max: 60 scale 10
- Find all local maxima.
- Find all local minima.
- Find the following values: $f(-1), f(2), f(0), f(.125)$
- Graph the following two functions and find their points of intersection using the intersect command on your calculator.

A local maximum or local minimum is a point on the graph where there is a highest or lowest point within an interval such as the vertex of a parabola.

$$y = x^3 + 5x^2 - 7x + 2 \text{ and } y = 0.2x^2 + 10$$

Window: x min : -10 x max: 10 scale 1
y min: -10 y max: 50 scale 0

- Use a graphing calculator to determine which of the given viewing rectangles produces the most appropriate graph of the function $3f(x) = 10 + 25x - x^3$.
 - $[-4, 4]$ by $[-4, 4]$
 - $[-10, 10]$ by $[-10, 10]$
 - $[-20, 20]$ by $[-100, 100]$
 - $[-100, 100]$ by $[-200, 200]$