TIMOTHY
CHRISTIAN SCHOOL

Entering AP Calculus Summer Packet

All students approved for AP Calculus are to complete this packet as well as assignments posted to the AP Classroom. https://apclassroom.collegeboard.org *Students are to login to AP Classroom using the join code provided by your instructor. Use the AP Classroom resources for help as well as your Timothy Christian Calculus textbook which can be borrowed from Timothy Christian School over the summer. Unless told otherwise, this summer assignment will be due on the first day of class and will be graded on neatness, completeness and accuracy. Show all your logical steps in arriving at the solution. Do not list only answers.*

All answers and work need to be done on separate sheets of paper that are clearly labeled for each section and problem number. Write all answers on a final answer sheet.

Skills needed for AP Calculus

I. Algebra

A. Exponents (operations with integer, fractional, and negative exponents)

B. Factoring (GCF, trinomials, difference of squares and cubes, sum of cubes, grouping)

C. Rationalizing (numerator and denominator)

D. Simplifying rational expressions

E. Solving algebraic equations and inequalities

F. Simultaneous equations

II. Graphing Functions

A. Lines (intercepts, slopes, write equations using point slope and slope intercept form, parallel, perpendicular, distance and midpoint formulas)

B. Conic Sections (circle, parabola, ellipse)

C. Functions (definition, notation, domain, range, inverse, composition)

D. Basic shapes and transformations of the following functions (absolute value, rational, root, higher order curves, log, ln, exponential, trigonometric, piece wise, inverse functions)

E. Tests for symmetry: odd, even

III. Logarithmic and Exponential Functions

A. Simplify expressions

B. Solve exponential and logarithmic equations

C. Inverses

IV. Geometry and Trigonometry

A. Area and Perimeter

B. Unit Circle (definition of functions, angles in radians and degrees)Unit Circle

C. Trig Functions

D. Identities

E. Solving Trig Equations

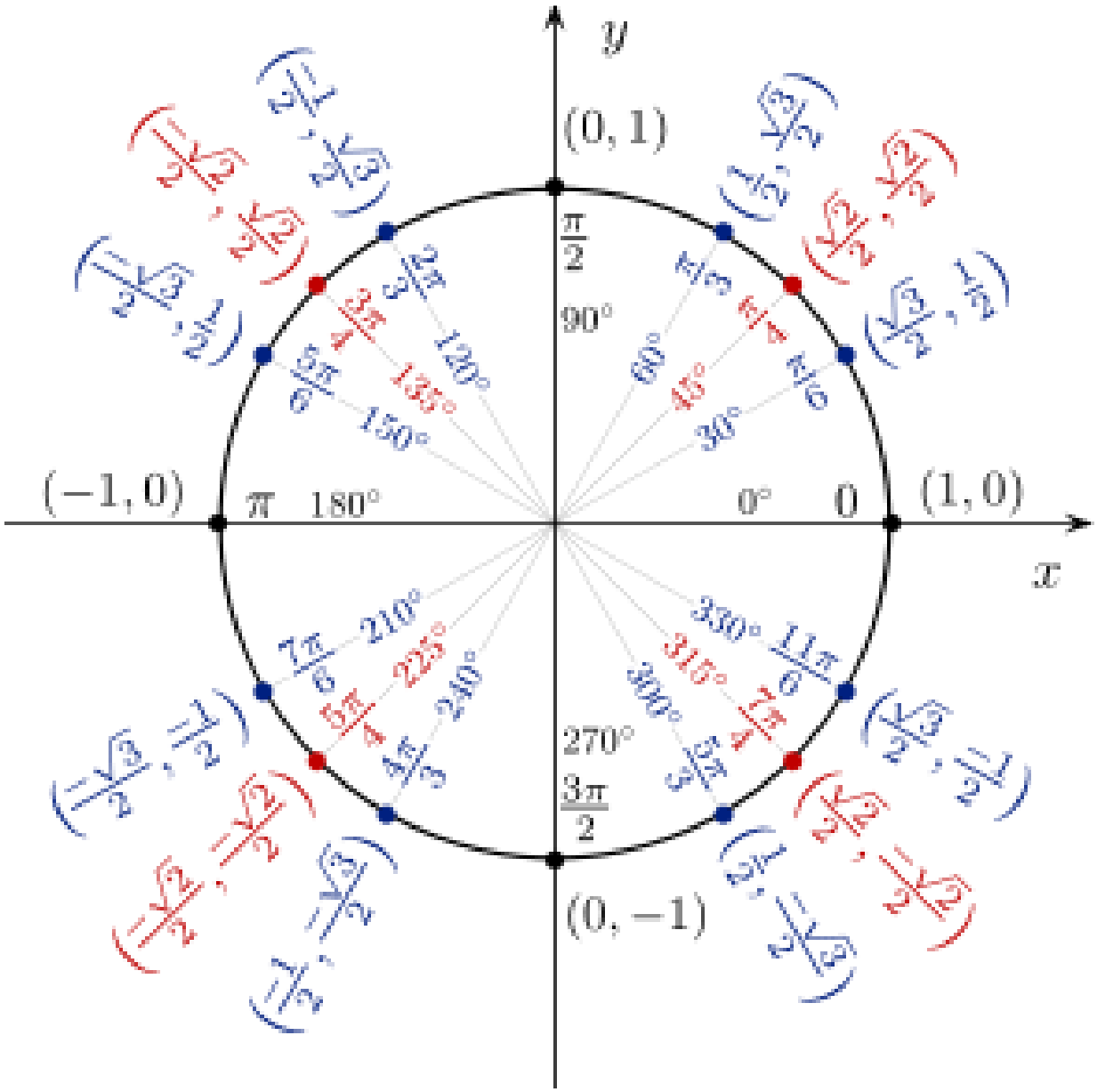
F. Inverse Trigonometric Functions

G. Right Triangle Trigonometry

V. Graphing Calculator

UNIT CIRCLE

RADIUS = 1 UNIT



Helpful Formulas

Trig Formulas

Arc Length of a circle: $L = r\theta \ \ or \ \ L = \frac{d}{360}(2\pi r)$

Area of a sector of a circle: $Area = \frac{1}{2}r^2\theta \quad or \quad Area = \frac{d}{360}(\pi r^2)$

Solving Parts of a Triangle

Law of Sines: $\frac{sinA}{a} = \frac{sinB}{b} = \frac{sinC}{c}$

Law of Cosine: $a^2 = b^2 + c^2 - 2bccosA$

$b^2 = a^2 + c^2 - 2accosB$

$c^2 = a^2 + b^2 - 2abcosC$

Area of a Triangle

$Area = \frac{1}{2}bcsinA \quad or \quad Area = \frac{1}{2}acsinB \quad or \quad Area = \frac{1}{2}absinC$

Herron's Formula $Area = \sqrt{s(s-a)(s-b)(s-c)} \ \ where \ s = \frac{1}{2}(a+b+c)$

Trig Identities

Reciprocal Identities: $cscA = \frac{1}{sinA} \quad secA = \frac{1}{cosA} \quad cotA = \frac{1}{tanA}$

Quotient Identities: $tanA = \frac{sinA}{cosA} \quad cotA = \frac{cosA}{sinA}$

Pythagorean Identities

$sin^2A + cos^2A = 1 \quad tan^2A + 1 = sec^2A \quad 1 + cot^2A = csc^2A$

Sum and Difference Identities

$\sin(A+B) = sinAcosB + cosAsinB \quad \sin(A-B) = sinAcosB - cosAsinB$

$\cos(A+B) = cosAcosB - sinAsinB \quad \cos(A-B) = cosAcosB + sinAsinB$

$\tan(A+B) = \frac{tanA + tanB}{1 - tanAtanB} \quad \tan(A+B) = \frac{tanA - tanB}{1 + tanAtanB}$

Double Angle Identities

$\sin(2A) = 2sinAcosA \quad \tan(2A) = \frac{2tanA}{1 - tan^2A}$

$\cos(2A) = cos^2A - sin^2A \quad \cos(2A) = 2cos^2A - 1 \quad \cos(2A) = 1 - 2sin^2A$

Half Angle Identities

$sin\frac{A}{2} = \pm\sqrt{\frac{1-cosA}{2}} \quad cos\frac{A}{2} = \pm\sqrt{\frac{1+cosA}{2}}$

Calculus Prerequisite Problems

I. Algebra

A. Simplify $\dfrac{(8x^3yz)^{\frac{1}{3}}(2x)^3}{4x^{\frac{1}{3}}(yz^{\frac{2}{3}})^{-1}}$

B. Factor Completely (Grouping, GCF, difference of squares or cubes)

1. $9x^2 + 3x - 3xy - y$

2. $64x^6 - 1$

3. $42x^4 + 35x^2 - 28$

4. $15x^{\frac{5}{2}} - 2x^{\frac{3}{2}} - 24x^{\frac{1}{2}}$ *(factor out* $x^{\frac{1}{2}}$ *first)*

C. Rationalize

1. $\dfrac{3 - x}{1 - \sqrt{x - 2}}$

2. $\dfrac{\sqrt{x + 1} + 1}{x}$

D. Simplfy the rational expression

$$\frac{(x + 1)^3(x - 2) + 3(x + 1)^2}{(x + 1)^4}$$

E. Solve algebraic equations and inequalities

1 – 2 ***Use synthetic division to help factor the following, state all factors and roots.***

1. $p(x) = x^3 + 4x^2 + x - 6$

2. $p(x) = 6x^3 - 17x^2 - 16x + 7$

3. *Explain why* $\dfrac{3}{2}$ *cannot be a root of* $f(x) = 4x^5 + cx^3 - dx + 5$*, where c and d are integers.* (*hint*: *You can look at the possible rational roots.*)

4. *Explain why* $f(x) = x^4 + 7x^2 + x - 5$ *must have a root in the interval* $[0, 1]$, $(0 \le x \le 1)$. *Check the graph and use signs of* $f(0)$ *and* $f(1)$ *to justify your answer.*

Solve: *You may use a graphing calculator to check solutions.*

5. $(x+3)^2 > 4$ 6. $\frac{x+5}{x-3} \leq 0$ 7. $3x^3 - 14x^2 - 5x \leq 0 \quad (factor)$

8. $x < \frac{1}{x}$ 9. $\frac{x^2-9}{x+1} \geq 0$ 10. $\frac{1}{x-1} + \frac{4}{x-6} > 0$

11. $x^2 < 4$ 12. $|2x+1| < \frac{1}{4}$

F. Solve the system. Solve the system algebraically and then check the solution by graphing each function and using your calculator to find the points of intersection.

1. $\begin{matrix} x - y + 1 = 0 \\ y - x^2 = -5 \end{matrix}$ 2. $\begin{matrix} x^2 - 4x + 3 = y \\ -x^2 + 6x - 9 = y \end{matrix}$

***II. Graphing and Functions*:**

A. Linear graphs: Write the equation of the line described below.

1. *Passes through the point* $(2, -1)$ *and has a slope of* $-\frac{1}{3}$
2. *Passes through the point* $(4, -3)$ *and is perpendicular to* $3x + 2y = 4$.
3. *Passes through* $(-1, -2)$ *and is parallel to* $y = \frac{3}{5}x - 1$

B. Conic Sections: Write the equation in standard form and identify the conic.

$4x^2 - 16x + 3y^2 + 24y + 52 = 0$

C. Functions: Find the domain and range of the following.

Note: domain restrictions – denominator $\neq 0$, *argument of a log or ln* > 0,
radicand of even index must be ≥ 0

range restrictions – reasoning, if all else fails, use graphing calculator

1. $y = \frac{3}{x-2}$ 2. $\log(x-3)$ 3. $y = x^4 + x^2 + 2$

4. $y = \sqrt{2x-3}$ 5. $y = |x-5|$ 6. $y = \frac{\sqrt{x+1}}{x^2-1}$ *domain only*

7. *Given* $f(x)$ *below, graph over the domain* $[-3, 3]$, *what is the range?*

$$f(x) = \begin{cases} x & if\ x \geq 0 \\ 1 & if\ -1 \leq x < 0 \\ x - 2 & x < -1 \end{cases}$$

Find the composition /inverse as indicated below.

Let $f(x) = x^2 + 3x - 2$ $\quad g(x) = 4x - 3$ $\quad h(x) = lnx$ $\quad w(x) = \sqrt{x-4}$

8. $g^{-1}(x)$ 9. $h^{-1}(x)$ 10. $w^{-1}(x), for\ x \geq 4$ 11. $f(g(x))$ 12. $h(g(f(1)))$

13. *Does* $y = 3x^2 - 9$ *have an inverse function? Explain your answer.*

Let $f(x) = 2x$, $\ g(x) = -x$, $\ and\ h(x) = 4, find$

14. $(f\ o\ g)(x)$ 15. $(f\ o\ g\ o\ h)(x)$

16. *Let* $s(x) = \sqrt{4-x}$ *and* $t(x) = x^2, find\ the\ domain\ and\ range\ of\ (s\ o\ t)(x)$.

D. Basic Shapes of Curves:

Sketch the graphs. You may use your graphing calculator to verify your graph, but you should be able to graph the following by knowledge of the shape of the curve, by plotting a few points, and by your knowledge of transformations.

1. $y = \sqrt{x}$ 2. $y = lnx$ 3. $y = \frac{1}{x}$ 4. $y = |x - 2|$ 5. $y = \frac{1}{x-2}$

6. $y = \frac{x}{x^2-4}$ 7. $y = 2^{-x}$ 8. $y = 3sin2(x - \frac{\pi}{6})$

9. $f(x) = \begin{cases} \sqrt{25-x^2} & if\ x < 0 \\ \frac{x^2-25}{x-5} & if\ x \geq 0,\ x \neq 5 \\ 0 & if\ x = 5 \end{cases}$

E. Even, Odd, Tests for Symmetry:

Identify as odd, even , or neither and justify you're answer. To justify your answer you must show substitution using $-x$ *! It is not enough to simply check a number.*

Even: if $f(x) = f(-x)$ $\quad$ *Odd: if* $f(-x) = -f(x)$

1. $f(x) = x^3 + 3x$ 2. $f(x) = x^4 - 6x^2 + 3$ 3. $f(x) = \frac{x^3 - x}{x^2}$

4. $f(x) = sin\ 2x$ 5. $f(x) = x^2 + x$ 6. $f(x) = x(x^2 - 1)$

7. $f(x) = \dfrac{1 + |x|}{x^2}$

8. *What type of function (even or odd) results from the product of two even functions? Two odd functions?*

Test for symmetry. Show substitution with variables to justify you're answer.

→ *Symmetric to y axis: replace x with* $-$ *x and relation remains the same.*

→ *Symmetric to x axis: replace y with* $-$ *y and relation remains the same.*

→ *Origin symmetry: replace x with* $-$ *x, y with* $-$ *y and the relation is equivalent.*

1. $y = x^4 + x^2$ 2. $y = sin(x)$ 3. $y = cos(x)$

4. $x = y^2 + 1$ 5. $y = \dfrac{|x|}{x^2 + 1}$

III. Logarithmic and Exponential Functions

A. Simplify expressions:

1. $log_4\left(\frac{1}{16}\right)$ 2. $3log_3 3 - \frac{3}{4}log_3 81 + \frac{1}{3}log_3\left(\frac{1}{27}\right)$ 3. $log_9 27$

4. $log_{125}\left(\frac{1}{5}\right)$ 5. $log_w w^{45}$ 6. $ln\, e$ 7. $\ln 1$ 8. lne^2

B. Solve equations:

1. $log_6(x + 3) + log_6(x + 4) = 1$ 2. $log\, x^2 - log\, 100 = log\, 1$ 3. $3^{x+1} = 15$

C. *Find the inverse for each function.*

a. $f(x) = \sqrt{10 - 3x}$

b. $f(x) = e^{x^3}$

c. $y = \ln(x + 3)$

IV. Geometry and Trigonometry

A. Area and Perimeter

1. *A rectangle has perimeter* 20 *m. Express the area of the rectangle as a function of the length of one of its sides.*

2. Find the area of a 30 degree sector in a circle with radius equal to 5 inches.

B. Unit Circle: Know the unit circle – radian and degree measure. Be prepared for a quiz.

C. Trigonometric Functions

1. *State the domain, range and period for each function?*

a) $y = \sin x$ *b)* $y = \cos x$ *c)* $y = \tan x$

D. Identities:

Simplify: 1. $\dfrac{(\tan^2 x)(\csc^2 x) - 1}{(\csc x)(\tan^2 x)(\sin x)}$ 2. $1 - \cos^2 x$ 3. $\sec^2 x - \tan^2 x$

Verify: 4. $(1 - \sin^2 x)(1 + \tan^2 x) = 1$

E. Solve the Equations

1. $\cos^2 x = \cos x + 2, \quad 0 \le x \le 2\pi$ 2. $2\sin(2x) = \sqrt{3}, \quad 0 \le x \le 2\pi$

3. $\cos^2 x + \sin x + 1 = 0, \quad 0 \le x \le 2\pi$

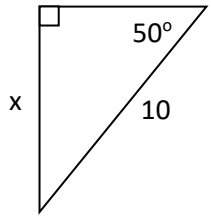
F. Inverse Trig Functions: Note: $\sin^{-1} x = \arcsin x$

1. $\arcsin 1$ 2. $\arcsin\left(-\dfrac{\sqrt{2}}{2}\right)$ 3. $\arccos\left(\dfrac{\sqrt{3}}{2}\right)$ 4. $\sin\left(\arccos\left(\dfrac{\sqrt{3}}{2}\right)\right)$

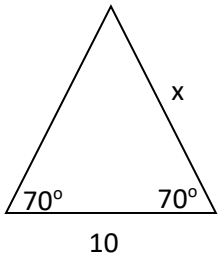
5. *State domain and range for*: $\arcsin(x)$, $\arccos(x)$, $\arctan(x)$

G. Right Triangle Trig: Find the value of x. (*Note: Degree measure*!)

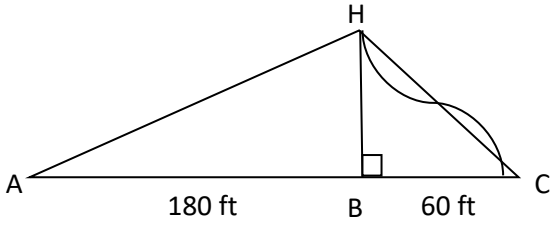
1.



2.



3.



3. *The roller coaster car shown in the diagram above takes* 23.5 *sec. to go up the* 23 *degree incline segment AH and only* 2.8 *seconds to go down the drop from H to C. The car covers horizontal distances of* 180 *feet on the incline and* 60 *feet on the drop. Decimals in answer may vary.*

a. How high is the roller coaster above point B?

b. Find the distances AH and HC.

c. How fast (in ft/sec) does the car go up the incline?

d. What is the approximate average speed of the car as it goes down the drop?

e. Assume the car travels along HC. Is your approximate answer too big or too small?

V. Graphing Calculator:

Be familiar with the CALC *commands; value, root, minimum, maximum, intersect. You may need to zoom in on areas of your graph to find the information.*

Answers should be accurate to 3 *decimal places. Sketch graph.*

1 – 4 *Given the following function* $f(x) = 2x^4 - 11x^3 - x^2 + 30x$.

1. *Find all roots.* *Note: Window x min:* -10 *x max:* 10 *scale* 1 *y min:* -100 *y max:* 60 *scale* 10
2. *Find all local maxima.*
3. *Find all local minima.*
4. *Find the following values:* $f(-1), f(2), f(0), f(.125)$

A local maximum or local minimum is a point on the graph where there is a highest or lowest poin within an interval such as the vertex of a parabola.

5. *Graph the following two functions and find their points of intersection using the intersect command on your calculator.*

$y = x^3 + 5x^2 - 7x + 2$ *and* $y = 0.2x^2 + 10$ *Window: x min :* -10 *x max:* 10 *scale* 1 *y min:* -10 *y max:* 50 *scale* 0

6. *Use a graphing calculator to determine which of the given viewing rectangles produces the most appropriate graph of the function* $3f(x) = 10 + 25x - x^3$.

a. $[-4,4]$ *by* $[-4,4]$

b. $[-10,10]$ *by* $[-10,10]$

c. $[-20,20]$ *by* $[-100,100]$

d. $[-100,100]$ *by* $[-200,200]$