Entering AP Calculus Summer Packet

The AP Calculus course

The AP Calculus Course is a college level course coverring material traditionally taught in the first semester of college calculus. Students need a strong foundation of Algebra and Trigonometry to be ready for the rigorous work required throughout the AP Calculus course. Completing the prerequisite packet should help prepare you for the material taught in this course.

This packet will be collected on the first day of class and your grade will be determined on neatness, completeness of solutions and accuracy. In preparation for the AP test, students need to begin showing all work and logical steps. ***Do not list only your answers.***

In addition to this packet, students will need to complete assignments from Chapter 1 of the AP Calculus textbook. The textbook will need to be rented out through the front office for the summer.

All answers and work need to be done on separate sheets of paper that are clearly labeled for each section and problem number. Write all answers on a final answer sheet.

Skills needed for AP Calculus

I. Algebra

A. Exponents (operations with integer, fractional, and negative exponents)

B. Factoring (GCF, trinomials, difference of squares and cubes, sum of cubes, grouping)

C. Rationalizing (numerator and denominator)

D. Simplifying rational expressions

E. Solving algebraic equations and inequalities

F. Simultaneous equations

II. Graphing Functions

A. Lines (intercepts, slopes, write equations using point slope and slope intercept form, parallel, perpendicular, distance and midpoint formulas)

B. Conic Sections (circle, parabola, ellipse)

C. Functions (definition, notation, domain, range, inverse, composition)

D. Basic shapes and transformations of the following functions (absolute value, rational, root, higher order curves, log, ln, exponential, trigonometric, piece wise, inverse functions)

E. Tests for symmetry: odd, even

II. Geometry

A. Pythagorean Theorem

B. Area Formulas

C. Volume Formulas

D. Similar Triangles

IV. Logarithmic and Exponential Functions

A. Simplify expressions

B. Solve exponential and logarithmic equations

C. Sketch graphs

D. Inverses

V. Trigonometry

A. Unit Circle (definition of functions, angles in radians and degrees)

B. Use of Pythagorean identities and formulas to simplify expressions and prove identities

C. Solve equations

D. Inverse Trigonometric Functions

E. Right Triangle Trigonometry

F. Graphs

VI. Limits (Chapter 12 of PreCalc)

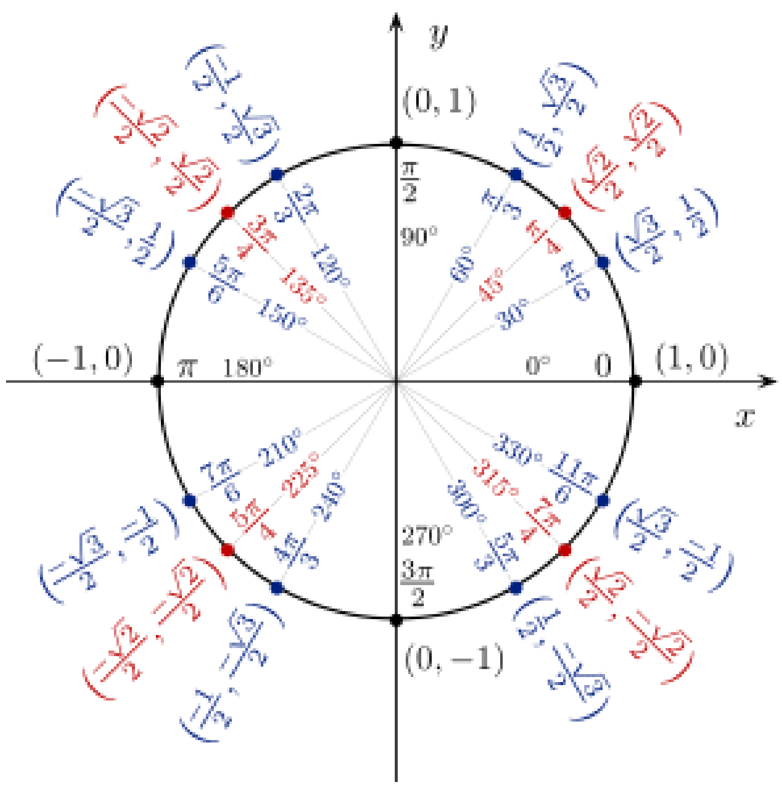
A. Concept of a limit

B. Find limits as x approaches a number and as x approaches ∞.

C. Secant and Tangent lines

D. Velocity as the slope of a tangent line

E. Derivatives and Integrals



<u>Helpful Formulas</u>

Trig Formulas

Arc Length of a circle: $L = r\theta \text{ or } L = \frac{d}{360}(2\pi r)$

Area of a sector of a circle: $Area = \frac{1}{2}r^2\theta \quad or \quad Area = \frac{d}{360}(\pi r^2)$

Solving Parts of a Triangle

Law of Sines: $\frac{sinA}{a} = \frac{sinB}{b} = \frac{sinC}{c}$

Law of Cosine: $a^2 = b^2 + c^2 - 2bccosA$

$b^2 = a^2 + c^2 - 2accosB$

$c^2 = a^2 + b^2 - 2abcosC$

Area of a Triangle

$Area = \frac{1}{2}bcsinA \quad or \quad Area = \frac{1}{2}acsinB \quad or \quad Area = \frac{1}{2}absinC$

Herron's Formula $Area = \sqrt{s(s-a)(s-b)(s-c)}$ *where* $s = \frac{1}{2}(a+b+c)$

Trig Identities

Reciprocal Identities: $cscA = \frac{1}{sinA} \quad secA = \frac{1}{cosA} \quad cotA = \frac{1}{tanA}$

Quotient Identities: $tanA = \frac{sinA}{cosA} \quad cotA = \frac{cosA}{sinA}$

Pythagorean Identities

$sin^2A + cos^2A = 1 \quad tan^2A + 1 = sec^2A \quad 1 + cot^2A = csc^2A$

Sum and Difference Identities

$\sin(A+B) = sinAcosB + cosAsinB \quad \sin(A-B) = sinAcosB - cosAsinB$

$\cos(A+B) = cosAcosB - sinAsinB \quad \cos(A-B) = cosAcosB + sinAsinB$

$\tan(A+B) = \frac{tanA + tanB}{1 - tanAtanB} \quad \tan(A+B) = \frac{tanA - tanB}{1 + tanAtanB}$

Double Angle Identities

$\sin(2A) = 2sinAcosA \quad \tan(2A) = \frac{2tanA}{1 - tan^2A}$

$\cos(2A) = cos^2A - sin^2A \quad \cos(2A) = 2cos^2A - 1 \quad \cos(2A) = 1 - 2sin^2A$

Half Angle Identities

$sin\frac{A}{2} = \pm\sqrt{\frac{1 - cosA}{2}} \quad cos\frac{A}{2} = \pm\sqrt{\frac{1 + cosA}{2}}$

Calculus Prerequisite Problems

I. Algebra

$A.\ Simplify\ \dfrac{(8x^3yz)^{\frac{1}{3}}(2x)^3}{4x^{\frac{1}{3}}(yz^{\frac{2}{3}})^{-1}}$

$B.\ Factor\ Completely\ (Grouping,\ GCF,\ difference\ of\ squares\ or\ cubes)$

1. $9x^2 + 3x - 3xy - y$

2. $64x^6 - 1$

3. $42x^4 + 35x^2 - 28$

4. $15x^{\frac{5}{2}} - 2x^{\frac{3}{2}} - 24x^{\frac{1}{2}}\ (factor\ out\ x^{\frac{1}{2}}\ first)$

$C.\ Rationalize$

1. $\dfrac{3 - x}{1 - \sqrt{x - 2}}$

2. $\dfrac{\sqrt{x + 1} + 1}{x}$

$D.\ Simplfy\ the\ rational\ expression$

$\dfrac{(x + 1)^3(x - 2) + 3(x + 1)^2}{(x + 1)^4}$

$E.\ Solve\ algebraic\ equations\ and\ inequalities$

1 – 2 $\boldsymbol{Use\ synthetic\ division\ to\ help\ factor\ the\ following,\ state\ all\ factors\ and\ roots.}$

1. $p(x) = x^3 + 4x^2 + x - 6$

2. $p(x) = 6x^3 - 17x^2 - 16x + 7$

3. $Explain\ why\ \dfrac{3}{2}\ cannot\ be\ a\ root\ of\ f(x) = 4x^5 + cx^3 - dx + 5, where\ c\ and\ d\ are\ integers.$
$(hint: You\ can\ look\ at\ the\ possible\ rational\ roots.)$

4. $Explain\ why\ f(x) = x^4 + 7x^2 + x - 5\ must\ have\ a\ root\ in\ the\ interval\ [0, 1],\ (0 \le x \le 1).$
$Check\ the\ graph\ and\ use\ signs\ of\ f(0) and\ f(1) to\ justify\ your\ answer.$

$Solve: You\ may\ use\ a\ graphing\ calculator\ to\ check\ solutions.$

5. $(x + 3)^2 > 4$

6. $\dfrac{x + 5}{x - 3} \le 0$

7. $3x^3 - 14x^2 - 5x \le 0\quad (factor)$

8. $x < \frac{1}{x}$

9. $\frac{x^2 - 9}{x + 1} \geq 0$

10. $\frac{1}{x - 1} + \frac{4}{x - 6} > 0$

11. $x^2 < 4$

12. $|2x + 1| < \frac{1}{4}$

F. Solve the system. Solve the system algebraically and then check the solution by graphing each function and using your calculator to find the points of intersection.

1. $\begin{aligned} x - y + 1 = 0 \\ y - x^2 = -5 \end{aligned}$

2. $\begin{aligned} x^2 - 4x + 3 = y \\ -x^2 + 6x - 9 = y \end{aligned}$

***II. Graphing and Functions*:**

A. Linear graphs: Write the equation of the line described below.

1. *Passes through the point* $(2, -1)$ *and has a slope of* $-\frac{1}{3}$

2. *Passes through the point* $(4, -3)$ *and is perpendicular to* $3x + 2y = 4$.

3. *Passes through* $(-1, -2)$ *and is parallel to* $y = \frac{3}{5}x - 1$

B. Conic Sections: Write the equation in standard form and identify the conic.

1. $x = 4y^2 + 8y - 3$

2. $4x^2 - 16x + 3y^2 + 24y + 52 = 0$

C. Functions: Find the domain and range of the following.

Note: domain restrictions − denominator ≠ 0, argument of a log or ln > 0,

radicand of even index must be ≥ 0

range restrictions − reasoning, if all else fails, use graphing calculator

1. $y = \frac{3}{x - 2}$

2. $\log(x - 3)$

3. $y = x^4 + x^2 + 2$

4. $y = \sqrt{2x - 3}$

5. $y = |x - 5|$

6. $y = \frac{\sqrt{x+1}}{x^2-1}$ *domain only*

7. *Given* $f(x)$ *below, graph over the domain* $[-3, 3]$, *what is the range?*

$$f(x) = \begin{cases} x & \text{if } x \geq 0 \\ 1 & \text{if } -1 \leq x < 0 \\ x - 2 & x < -1 \end{cases}$$

Find the composition / inverse as indicated below.

Let $f(x) = x^2 + 3x - 2$ $\quad g(x) = 4x - 3$ $\quad h(x) = \ln x$ $\quad w(x) = \sqrt{x - 4}$

8. $g^{-1}(x)$

9. $h^{-1}(x)$

10. $w^{-1}(x)$, *for* $x \geq 4$

11. $f(g(x))$

12. $h(g(f(1)))$

13. Does $y = 3x^2 - 9$ have an inverse function? Explain your answer.

Let $f(x) = 2x$, $g(x) = -x$, and $h(x) = 4$, find

14. $(f \text{ o } g)(x)$ 15. $(f \text{ o } g \text{ o } h)(x)$

16. Let $s(x) = \sqrt{4 - x}$ and $t(x) = x^2$, find the domain and range of $(s \text{ o } t)(x)$.

D. Basic Shapes of Curves:

Sketch the graphs. You may use your graphing calculator to verify your graph, but you should be able to graph the following by knowledge of the shape of the curve, by plotting a few points, and by your knowledge of transformations.

1. $y = \sqrt{x}$ 2. $y = \ln x$ 3. $y = \frac{1}{x}$ 4. $y = |x - 2|$ 5. $y = \frac{1}{x - 2}$

6. $y = \frac{x}{x^2 - 4}$ 7. $y = 2^{-x}$ 8. $y = 3\sin 2(x - \frac{\pi}{6})$

9. $f(x) = \begin{cases} \sqrt{25 - x^2} & \text{if } x < 0 \\ \dfrac{x^2 - 25}{x - 5} & \text{if } x \geq 0,\ x \neq 5 \\ 0 & \text{if } x = 5 \end{cases}$

E. Even, Odd, Tests for Symmetry:

Identify as odd, even , or neither and justify you're answer. To justify your answer you must show substitution using $-x$! It is not enough to simply check a number.

Even: if $f(x) = f(-x)$	Odd: if $f(-x) = -f(x)$

1. $f(x) = x^3 + 3x$ 2. $f(x) = x^4 - 6x^2 + 3$ 3. $f(x) = \frac{x^3 - x}{x^2}$

4. $f(x) = \sin 2x$ 5. $f(x) = x^2 + x$ 6. $f(x) = x(x^2 - 1)$

7. $f(x) = \frac{1 + |x|}{x^2}$

8. What type of function (even or odd) results from the product of two even functions? Two odd functions?

$Test\ for\ symmetry.\ \ Show\ substitution\ with\ variables\ to\ justify\ you're\ answer.$

$\rightarrow\ Symmetric\ to\ y\ axis: replace\ x\ with\ -x\ and\ relation\ remains\ the\ same.$

$\rightarrow\ Symmetric\ to\ x\ axis: replace\ y\ with\ -y\ and\ relation\ remains\ the\ same.$

$\rightarrow\ Origin\ symmetry: replace\ x\ with\ -x, y\ with\ -y\ and\ the\ relation\ is\ equivalent.$

1. $y = x^4 + x^2$ 2. $y = sin(x)$ 3. $y = cos(x)$

4. $x = y^2 + 1$ 5. $y = \frac{|x|}{x^2 + 1}$

$\boldsymbol{IV.\ Logarithmic\ and\ Exponential\ Functions}$

$A.\ Simplify\ expressions:$

1. $log_4\left(\frac{1}{16}\right)$ 2. $3log_3 3 - \frac{3}{4}log_3 81 + \frac{1}{3}log_3\left(\frac{1}{27}\right)$ 3. $log_9 27$

4. $log_{125}\left(\frac{1}{5}\right)$ 5. $log_w w^{45}$ 6. $ln\ e$ 7. $\ln 1$ 8. lne^2

$B.\ Solve\ equations:$

1. $log_6(x + 3) + log_6(x + 4) = 1$ 2. $log\ x^2 - log\ 100 = log\ 1$ 3. $3^{x+1} = 15$

$\boldsymbol{V.\ Trigonometry}$

$A.\ \ Unit\ Circle: Know\ the\ unit\ circle - radian\ and\ degree\ measure. Be\ prepared\ for\ a\ quiz.$

1. $State\ the\ domain, range\ and\ fundamental\ period\ for\ each\ function?$

$a)\ y = sin\ x$ $b)\ y = cos\ x$ $c)\ y = tan\ x$

$B. Identities:$

$Simplify:$ 1. $\frac{(tan^2x)(csc^2x) - 1}{(cscx)(tan^2x)(sinx)}$ 2. $1 - cos^2 x$ 3. $sec^2x - tan^2x$

$Verify:$ 4. $(1 - sin^2x)(1 + tan^2x) = 1$

$C.\ Solve\ the\ Equations$

1. $cos^2x = cos\ x + 2, \quad 0 \le x \le 2\pi$ 2. $2\ sin(2x) = \sqrt{3}, \quad 0 \le x \le 2\pi$

3. $cos^2x + sinx + 1 = 0, \quad 0 \le x \le 2\pi$

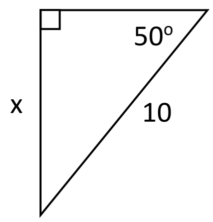
$D. Inverse\ Trig\ Functions: Note:\ \ Sin^{-1}x = Arcsin\ x$

1. $Arcsin1$ 2. $Arcsin\left(-\frac{\sqrt{2}}{2}\right)$ 3. $Arccos\left(\frac{\sqrt{3}}{2}\right)$ 4. $sin\left(Arccos\left(\frac{\sqrt{3}}{2}\right)\right)$

5. *State domain and range for:* $Arcsin(x)$, $Arccos\ (x)$, $Arctan\ (x)$

E. *Right Triangle Trig: Find the value of* x. *(Note: Degree measure!)*

1.



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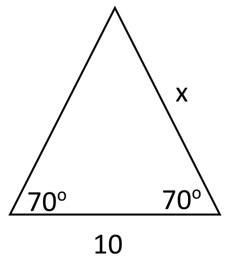
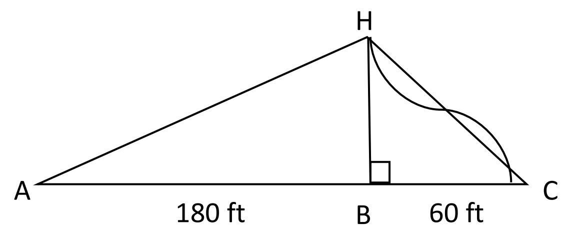


3.



3. *The roller coaster car shown in the diagram above takes* 23.5 *sec. to go up the* 23 *degree incline segment AH and only* 2.8 *seconds to go down the drop from H to C. The car covers horizontal distances of* 180 *feet on the incline and* 60 *feet on the drop. Decimals in answer may vary.*

a. *How high is the roller coaster above point B?*

b. *Find the distances AH and HC.*

c. *How fast (in ft/sec) does the car go up the incline?*

d. *What is the approximate average speed of the car as it goes down the drop?*

e. *Assume the car travels along HC. Is your approximate answer too big or too small?*

F. *Graphs: Identify the amplitude, period, horizontal, and vertical shifts of these functions.*

1. $y = -2\sin(2x)$ 2. $y = -\pi\cos\left(\frac{\pi}{2}x + \pi\right)$

G. *Be able to do the following on your graphing calculator:*

Be familiar with the CALC *commands; value, root, minimum, maximum, intersect. You may need to zoom in on areas of your graph to find the information.*

Answers should be accurate to 3 *decimal places. Sketch graph.*

1 – 4 *Given the following function* $f(x) = 2x^4 - 11x^3 - x^2 + 30x$.

1. *Find all roots.* *Note: Window x min:* -10 *x max:* 10 *scale* 1 *y min:* -100 *y max:* 60 *scale* 10
2. *Find all local maxima.*
3. *Find all local minima.*
4. *Find the following values:* $f(-1), f(2), f(0), f(.125)$

A local maximum or local minimum is a point on the graph where there is a highest or lowest poin within an interval such as the vertex of a parabola.

5. *Graph the following two functions and find their points of intersection using the intersect command on your calculator.*

$y = x^3 + 5x^2 - 7x + 2$ *and* $y = .2x^2 + 10$

Window: $x\ min: -10\ x\ max: 10\quad scale\ 1$

$y\ min: -10\ y\ max: 50\ scale\ 0$

VI. Functions and Models

1. *The graphs of f and g are given.*

(a) *State the values of* $f(-4)$ *and* $g(3)$.

(b) *For what values of x if* $f(x) = g(x)$?

(c) *Estimate the solution of the equation* $f(x) = -1$.

(d) *On what interval is f decreasing?*

(e) *State the domain and range of f.*

(f) *State the domain and range of g.*

2. *The number N (in thousands) of cellular phone subscribers in Malaysia is shown in the table. (Midyear estimates are given.)*

t	1991	1993	1995	1997
N	132	304	873	2461

(a) *Use the data to sketch a rough graph of N as a function of t.*

(b) *Use your graph to estimate the number of cell – phone subscribers in Malaysia at midyear in 1994 and in 1996.*

3. *If* $f(x) = 3x^2 - x + 2$, *find* $f(2)$, $f(-2)$, $f(a)$, $f(-a)$, $f(a+1)$, $2f(a)$, $f(a\ )$, $[f(a)]2$, $f(a+h)$.

4. *Find the domain of each function.*

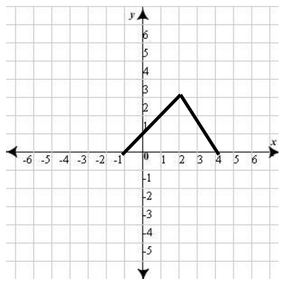
a. $f(x) = \dfrac{x}{3x-1}$

b. $g(u) = \sqrt{u} + \sqrt{4-u}$

5. *Find an expression for the bottom half of the parabola* $x + (y-1)^2 = 0$

6. *A rectangle has perimeter 20 m. Express the area of the rectangle as a function of the length of one of its sides.*

7. *Find an expression for the function whose graph is the given curve. Hint: peicewise*



8. *Biologists have noticed that the chirping rate of crickets of a certain species is related to temperature, and the relationship appears to be very nearly linear. A cricket produces* 113 *chirps per minute at* 70° *F and* 173 *chirps per minute at* 80° *F.*

a. *Find a linear equation that models the temperature T as a function of the number of chirps per minute N.*

b. *What is the slope of the graph? What does it represent?*

c. *If the crickets are chirping at* 150 *chirps per minute, estimate the temperature.*

9. *At the surface of the ocean, the water pressure is the same as the air pressure above the water,* 15 lb/in^2 *Below the surface, the water pressure increases by* 4.34 lb/in^2 *for every* 10 *ft of descent.*

a. *Express the water pressure as a function of the depth below the ocean surface.*

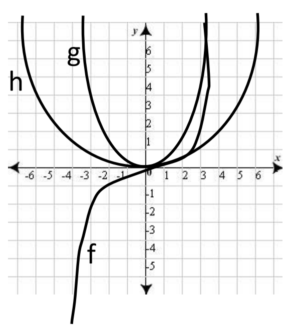
b. *At what depth is the pressure* 100 lb/in ? 2

10. *Classify each function as a power function, root function, polynomial (state its degree), rational function, algebraic function, or logarithmic function.*

a. $f(x) = \sqrt[5]{x}$ b. $g(x) = \sqrt{1 - x^2}$ c. $h(x) = x^9 + x^4$ d. $r(x) = \dfrac{x^2 + 1}{x^3 + x}$

11. *Match each equation with its graph. Explain your choices. (Don't use a computer or graphing calculator).*

a. $y = x^2$ b. $y = x^5$ c. $y = x^8$

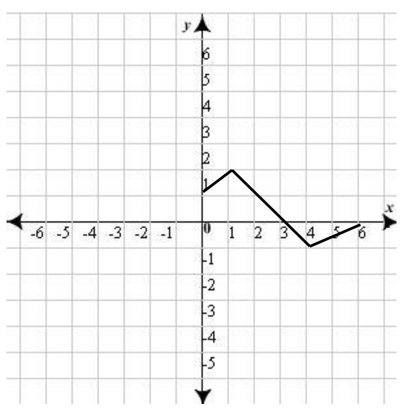


12. *Suppose the graph of f is given. Write equations for the graphs that are obtained from the graph of f as follows.*

a. *Shift* 3 *units upward.*
b. *Shift* 3 *units downward.*
c. *Shift* 3 *units to the right.*
d. *Shift* 3 *units to the left.*
e. *Reflect about the* $x-$*axis.*
f. *Reflect about the* $y-$*axis.*
g. *Stretch vertically by a factor of* 3.
h. *Shrink vertically by a factor of* 3.

14. *The graph of f is given. Use it to graph the following functions.*

a. $y = f(2x)$ b. $y = f(\frac{1}{2}x)$ c. $y = f(-x)$ d. $y = -f(-x)$



15. *Graph the following, not by plotting points, but by starting with the graph of one of the parent function and applying the appropriate transformations.*

$$y = \frac{1}{3} sin\left(x - \frac{\pi}{6}\right)$$

16. *Find* $f + g$, $f - g$, fg, *and* $\frac{f}{g}$ *and state their domains.*

$f(x) = x^3 + 2x^2$ $g(x) = g(x) = 3x^2 - 1$

17. *Find the functions* $f\ o\ g$, $g\ o\ f$, $f\ o\ f$, *and* $g\ o\ g$ *and their domains.*

$f(x) = sinx$ $g(x) = 1 - \sqrt{x}$

18. *Express the function* $F(x)$ *in the form* $f\ o\ g$.

$F(x) = (x^2 + 1)^{10}$

$19.$ *Use a graphing calculator to determine which of the given viewing rectangles produces the most appropriate graph of the function* $3\, f(\,x) = 10 + 25x - x^3$.

$a.$ $[-4,4\}$ *by* $[-4,4]$

$b.$ $[-10,10]$ *by* $[-10,10]$

$c.$ $[-20,20]$ *by* $[-100, 100]$

$d.$ $[-100, 100]$*by* $[-200,200]$

$20.$ *Find the inverse for each function.*

$a.$ $f(x) = \sqrt{10 - 3x}$

$b.$ $f(x) = e^{x^3}$

$c.$ $y = \ln(x + 3)$

$21.$ *Find the exact value of each expression* (*no calculator*)

$a.$ $log_2 64$ $b.$ $log_6 \frac{1}{36}$ $c.$ $log_{10} 1.25 + log_{10} 80$ $d.$ $log_5 10 + log_5 20 - 3log_5 2$

$22.$ *Express the given quantity as a single logarithm.*

$a.$ $2ln4 - ln2$

VI. Limits

$1.$ *Evaluate each one sided limit.*

$a.$ $\lim_{x \to 0^+} \sqrt{x+4} - 8$ $b.$ $\lim_{x \to 4} \frac{x^2 - 25}{x - 5}$

$2.$ *Evaluate each limit if it exists.*

$a.$ $\lim_{x \to 7} \frac{6}{x - 7}$ $b.$ $\lim_{x \to \infty} x^3 + 5x^2 - 2x + 21$

$3.$ *The avergae cost C in dollars of x number of personal digital assistants can be modelled by*

$$C(x) = \frac{100x + 7105}{x}$$

$a.$ *Determine the limit of the function as x approaches infinity.*

$b.$ *Interpret the results from part a.*

$4.$ *Use direct substitution to find each limit.*

a. $\lim_{x \to 5} \frac{x^2}{\sqrt{x-4}-2}$ b. $\lim_{x \to 9} 2x^3 - 12x + 3$

5. *Evaulate each limit.*

a. $\lim_{x \to \infty} x^2 - 7x + 2$ b. $\lim_{x \to \infty} 2x^3 - 8x^2 - 5$

c. $\lim_{x \to \infty} \frac{2x^3 - x - 1}{-x^4 + 7x^3 + 4}$ d. $\lim_{x \to \infty} \frac{\sqrt{25+x}-4}{x}$

6. *Find the slope of the line tangent to the graph of each function at the given points.*

a. $y = x^2 + 2x - 8;\ (-5,7) and\ (-2,8)$

b. $y = \frac{4}{x^3} + 2;\ (-1,-2)\ and\ \left(2, \frac{5}{2}\right)$

7. *Find an equation for the instantaneous velocity $v(t)$ if the path of an object is defined as $h(t)$ for any point in time t.*

a. $h(t) = 9t + 3t^2$ b. $h(t) = 10t^2 - 7t^3$

8. *Find the derivative of each function.*

a. $f(x) = -3x - 7$

b. $b(c) = 4c^{\frac{1}{2}} - \ 8c^{\frac{2}{3}} + 5c^{\frac{4}{5}}$

c. $w(y) = 3y^{\frac{4}{3}} \mp 6y^{\frac{1}{4}}$

d. $g(x) = (x^2 - 4)(2x - 5)$

e. $h(t) = \frac{t^3 + 4t^2 + t}{t^2}$

Calculus of a Single Variable (Early Transcendental Functions)

I. 1.1 (page 2 – 6, Examples 1 – 5) Exercises: 1 – 4, 12, 24, 30, 31, 63, 64, 77
II. 1.2 (page 11 – 15, Examples 1 – 4) Exercises: 19, 60, 80, 83, 85, 97
III. 1.3 (page 16 – 26, Examples 1 – 5) Exercises: 11, 22, 23, 30, 57, 67, 99, 102
IV. 1.5 (page 37 – 43, Examples 1 – 7) Exercises: 1, 5, 9, 12, 17, 18, 26, 32, 45, 64, 80, 84, 96
V. 1.6 (page 49 – 5, Examples 1 – 6) Exercises: 2, 6, 9, 10, 18, 19, 25, 26, 35, 43 – 46, 71, 75